Electromagnetic Contributions to Lepton $g-2$ in a Thick Brane-World

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We consider Standard Model fields living inside a thick four dimensional flat brane embedded in a (possibly warped) five dimensional space-time and estimate the electromagnetic corrections to the anomalous magnetic moment $(g-2)$ of the electron and muon by including virtual massive fermion and gauge boson states. Constraints on the mass of the "excited" states (or thickness of the brane-world) are obtained.

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I. INTRODUCTION

Recently there has been a revived interest in models containing extra spatial dimensions. The first time this idea was put in concrete form possibly dates back to the '20s [1], when the existence of more than four dimensions was employed in an attempt to unify gravity with the electromagnetic (EM) field. In such an approach an electrically charged particle is extended in the fifth dimension which, because of the relative strength of gravitational and EM forces, is extremely small and as a consequence the charged particle is extremely massive (of order the Planck mass M_p).

Higher dimensional spaces also naturally come into play when strings are considered [2] and one must then compactify the fundamental theory down to our four dimensional world. In Refs. [3] a compactification scheme (whose low energy limit is eleven dimensional supergravity) was constructed for the strongly coupled $E_8 \times E_8$ heterotic string. Six dimensions are then compactified on a Calabi-Yau manifold and integrated out leaving a five dimensional (bulk) space-time bounded by two copies of the same D3-brane [4]. Matter particles are low energy excitations of open strings with end-points confined on the D3-brane, which would thus represent our (brane)world. Gravitons instead are closed strings and can propagate also in the extra dimension, which has topology S/Z_2. This construction yields a relation between the Planck mass on the D3-brane and the fundamental string mass scale which allows the latter to be much smaller than the former, hence suggesting a solution to the hierarchy problem.

Inspired by this result, several models have been proposed with various numbers of extra dimensions, which can be either compact [5] or infinitely extended with a warp factor [6]. In both cases, there are parameters which can be tuned to make the fundamental mass scale small enough to lead to new physics slightly above 1 TeV without violating Newton's law at the present level of confidence [7]. One of the main concerns in such models is to provide a (field theoretical model) confining mechanism for the matter fields which does not violate any of the tested properties of the Standard Model (SM) and yields, at the same time, predictable effects which can be probed by the forthcoming generation of detectors [5,8]. Early proposals for confining matter fields on a wall of codimension one are actually older and make use of the non-vanishing expectation value of a scalar field [9]. The fact that heavy ("excited") particle states living in the extra dimensions have not been detected yet is then generally a consequence of the small coupling between SM particles and bulk gravitons, namely $\mathcal{O}(1/M_p)$.

In the present letter, we shall consider a model in which the brane-world has finite thickness (of size $2L_f$ for fermions and $2L_b \geq 2L_f$ for bosons [10] which can be different, reflecting differing confining mechanisms). The five space-time coordinates are denoted by $\boldsymbol{x}$ (or Greek indices running from 0 to 3) for the usual space-time and y for the extra dimension. The metric inside the brane is flat Minkowski, $\eta_{\mu\nu} = \mathrm{diag}\,[-1,+1,+1,+1]$, and matches with an external (possibly warped [11]) space-time metric:

$$ds^2 = a^2(y)\,\eta_{\mu\nu}\,dx^\mu\,dx^\nu + dy^2 \ , \qquad (1)$$

where $a(y) = 1$ for $|y| < L_b$. The reason we allow for a warp factor is that we want to consider just one extra dimension. Further, the fields we shall study have support inside the (thick) brane where $a = 1$ and possible effects originating in the bulk are neglected here. For such a model, we estimate the (order of magnitude of) EM corrections to the anomalous magnetic moment of the SM electron and muon and, by comparing with the precision of the present measurements, obtain bounds for the mass of "excited" states which, of course, is related to the thickness of the brane.

II. EFFECTIVE FOUR-DIMENSIONAL ACTION

Fermion fields $\Psi = \Psi(\boldsymbol{x}, y)$ couple to the topological (kink) vacuum of a scalar field Φ [9], which we approximate as

$$\Phi = \begin{cases} -(m_f^2/2)\,L_f & y < -L_f \\ (m_f^2/2)\,y & |y| < L_f \\ +(m_f^2/2)\,L_f & y > +L_f \ . \end{cases} \qquad (2)$$

Therefore, as we review below, fermions have a confined massless (chiral) mode, together with a tower of

states which are allowed only if their mass is smaller than $m_f^2\, L_f/2 \equiv M_f$ [*].

On neglecting bulk contributions, the five dimensional action for fermions minimally coupled to gauge bosons in the brane is given by

$$S_{(5)} = \int_{-L_b}^{+L_b} a^4\, dy \int d^4x\, \bar{\Psi}\, \left(i\, \not{D} - \gamma^5\, \partial_y - \Phi \right)\, \Psi\ , \quad (3)$$

where $i\,\not{D} = -\not{p} + e\,\not{A}$, $\not{p} = -i\,\hbar\,\gamma^\mu\,\partial_\mu$ and e is the gauge coupling constant. Since $\gamma^5 = \Pi_L - \Pi_R$ (the difference between left and right chiral projectors), one can introduce "creation and annihilation" operators [10]

$$\hat{a}^\dagger = -\frac{1}{m_f}\, (\partial_y - \Phi)\ , \qquad \hat{a} = \frac{1}{m_f}\, (\partial_y + \Phi)\ , \quad (4)$$

such that $[\hat{a}, \hat{a}^\dagger] = 1$ and the Lagrangian density becomes

$$L_{(5)} = \bar{\Psi}\, \left(i\,\not{D} - m_f\, \hat{a}\, \Pi_L - m_f\, \hat{a}^\dagger\, \Pi_R \right)\, \Psi\ . \quad (5)$$

This allows an expansion for the fermions

$$\Psi(\boldsymbol{x}, y) = H_0(y)\, \Pi_L\, \psi^{(0)}(\boldsymbol{x}) + \sum_{n=1}^{N_f} \left[H_n(y)\, \Pi_L + H_{n-1}(y)\, \Pi_R \right] \psi^{(n)}(\boldsymbol{x})\ , \quad (6)$$

where H_n are the normalized eigenfunctions of the harmonic oscillator. Since the zero mode is massless,

$$(\not{p} + m_f\, \hat{a})\, H_0\, \Pi_L\, \psi^{(0)} = H_0\, \not{p}\, \Pi_L\, \psi^{(0)} = 0\ , \quad (7)$$

$\psi^{(0)}$ can be taken as a two-component Weyl spinor, $\psi^{(0)} = \Pi_L\, \psi^{(0)}$. We note in passing that $\hat{a}\, H_0 = 0$ is precisely the equation which ensures the confinement of the left zero mode within a width $\ell_f \sim 1/m_f$ around $y = 0$. Since for the (would-be) right zero mode the corresponding equation $\hat{a}^\dagger\, \bar{R}_0 = 0$ does not admit any (non-vanishing) normalizable solution in $y \in \mathbb{R}$, we have set $\Pi_R\, \psi^{(0)} = 0$.

The sum in Eq. (6) ends with a maximum integer $N_f < \infty$. The reason for such a cut-off can be easily understood if we set $\not{A} = 0$ and write down the Klein-Gordon equation corresponding to the Dirac equation obtained from $S_{(5)}$,

$$\begin{aligned} &\left(\not{p} - m_f\, \hat{a}^\dagger\, \Pi_L - m_f\, \hat{a}\, \Pi_R \right) \left(\not{p} + m_f\, \hat{a}\, \Pi_L + m_f\, \hat{a}^\dagger\, \Pi_R \right) \Psi \\ &= -\left(p^2 + m_f^2\, \hat{a}^\dagger\, \hat{a}\, \Pi_L + m_f^2\, \hat{a}\, \hat{a}^\dagger\, \Pi_R \right) \Psi \\ &= -\left[p^2 + \left(-\partial_y^2 + \Phi^2 \right) \right]\, \Psi = 0\ . \end{aligned} \quad (8)$$

It is thus clear that only those modes ψ_n whose eigenvalues $m_f^2\, n < \Phi^2(L) \equiv M_f^2$ can be retained (see Fig. 1) and, in the following, we shall consider only the simplest non-trivial case, that is the lowest level $n = 1$.

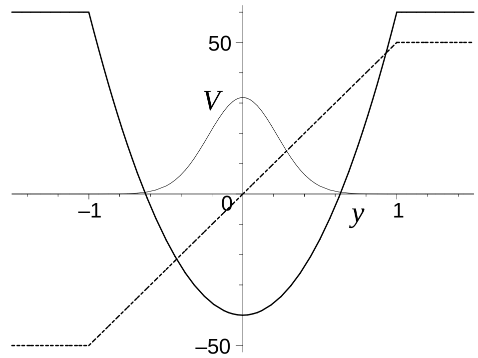

FIG. 1. Sketch of the scalar field Φ (magnified by a factor of 5; dashed line) and the corresponding confining potential in Eq. (8) (continuous line) for $m_f^2/2 = 10$ (in units with $L = 1$). The Gaussian curve represents the ground state H_0 (magnified by a factor of 10).

Since we need both chiralities to recover the correct low energy phenomenology, one doubles the fermion fields $\Psi \to (\Psi_1, \Psi_2)$ and pairs the two zero modes into one four-component Dirac fermion, $\psi^{(0)} = (\psi_1^{(0)}, C\, \psi_2^{(0)})$, where C denotes charge conjugation. Further, by introducing an interaction term of the form

$$L_m = m\, \bar{\psi}^{(0)}\, \psi^{(0)}\ , \quad (9)$$

the zero mode can be given the bare mass m, which arises as the vacuum expectation value of the Higgs field, and will have as (free) propagator

$$0\!-\!\!-\!\!-\!0 \quad = \frac{i}{\not{p} + m}\ . \quad (10)$$

The same mass term could be added for the $n \geq 1$ modes but would be negligible with respect to $n\, m_f$. Finally, one can assemble the left spinor components of the two generations of modes with $n = 1$ as $\psi^{(1)} = (\Pi_L\, \psi_1^{(1)}, C\, \Pi_L\, \psi_2^{(1)})$ [†], which will be propagated by

$$1\!-\!\!-\!\!-\!1 \quad = \frac{i}{\not{p} + m_f}\ . \quad (11)$$

Gauge bosons $A_\mu = A_\mu(\boldsymbol{x}, y)$ [‡] are assumed to be confined by an analogous mechanism to that of the fermions and, on neglecting edge effects, are parametrized by a tower of Kaluza-Klein (KK)-like states inside a "box" of size $2L_b$ [10] (in any case, we shall just consider the

[*] We shall not consider states whose energy exceeds the threshold of confinement.

[†] We omit writing the analogous rearrangement of the right components because it does not play any role in the process we are going to study, see Eq. (14).

[‡] It is always possible to set the fifth component $A_y = 0$ as a gauge choice.

lowest states which are not expected to depend on the detailed nature of the confining mechanism). Thus, one has a massless ground state and massive KK-like modes which are allowed provided their mass is smaller than the confining threshold M_b (possibly different from M_f)

$$
\begin{aligned}
A_\mu(\boldsymbol{x},y) = A_\mu^{(0)}(\boldsymbol{x}) + \sum_{n=1}^{N_b} B_\mu^{(n)}(\boldsymbol{x})\, \frac{\cos(\pi n y/L_b)}{\sqrt{L_b}} \\
+ \sum_{n=1}^{N_b} A_\mu^{(n)}(\boldsymbol{x})\, \frac{\sin(\pi n y/L_b)}{\sqrt{L_b}}\ ,
\end{aligned}
\tag{12}
$$

where both the modes $A_\mu^{(n)}$ and $B_\mu^{(n)}$ have an effective four dimensional mass $n\, m_b = \pi\, n/L_b \le M_b$. We shall again just keep the lowest level $n = 1$ and have the usual massless propagator for $A_\mu^{(0)}$ and the massive propagator for $A_\mu^{(1)}$ and $B_\mu^{(1)}$,

$$
\begin{aligned}
0 - - - 0 \quad &= \frac{i\, \eta_{\mu\nu}}{p^2} \\
1_A - - - 1_A &= \\
= 1_B - - - 1_B \quad &= i\, \frac{\eta_{\mu\nu} + p_\mu\, p_\nu/m_b^2}{p^2 + m_b^2}\ .
\end{aligned}
\tag{13}
$$

Upon inserting the expansion (6) for the two pairs of fermions and the gauge field (12) into the action $S_{(5)}$, just retaining the lowest levels, and integrating over the extra dimension, one obtains the effective four dimensional Lagrangian

$$
\begin{aligned}
L_{(4)} = \bar\psi^{(0)} &\left[\not p + m - e\, \not A^{(0)} - g_B^0\, \not B^{(1)}\right] \psi^{(0)} \\
&+ \bar\psi^{(1)} \left[\not p + m_f - e\, \not A^{(0)} - g_B^1\, \not B^{(1)}\right] \psi^{(1)} \\
&- g_A \left(\bar\psi^{(1)}\, \not A^{(1)}\, \psi^{(0)} + \bar\psi^{(0)}\, \not A^{(1)}\, \psi^{(1)}\right),
\end{aligned}
\tag{14}
$$

where we have made use of the parity and normalization properties of the functions H_n in (6) and sine and cosine in (12). We also obtain the five basic interaction vertices displayed in Fig. 2, where the effective gauge coupling constants depend on the overlap of the field modes in the extra dimension,

$$
\begin{aligned}
g_B^0 &= e \int H_0^2(y)\, \frac{\cos(m_b\, y)}{\sqrt{L_b}}\, dy \\
g_B^1 &= e \int H_1^2(y)\, \frac{\cos(m_b\, y)}{\sqrt{L_b}}\, dy \\
g_A &= e \int H_0(y)\, H_1(y)\, \frac{\sin(m_b\, y)}{\sqrt{L_b}}\, dy\ ,
\end{aligned}
\tag{15}
$$

and are plotted in Fig. 3 as functions of the ratio m_b/m_f which we expect to be related to L_f/L_b.

Consideration of additional levels ($n > 1$) will lead to analogous results and even more stringent constraints, that is larger masses for the allowed "excited" states. However, such contributions are expected to be even more dependent on the mechanism of confinement.

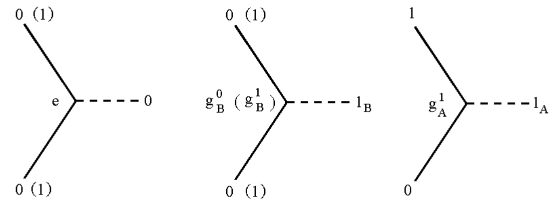

FIG. 2. Vertices of the effective theory for $N_f = N_b = 1$.

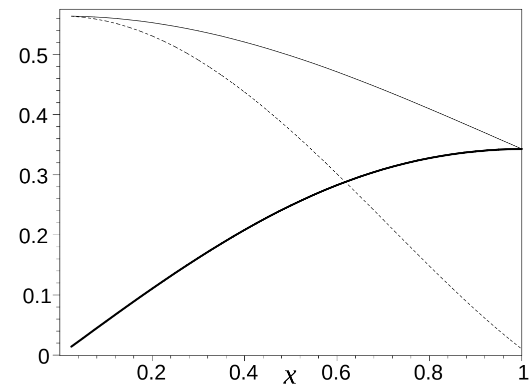

FIG. 3. Effective coupling constants (in units of e) as functions of $x \equiv m_b/m_f$: g_B^0 (thin solid line); g_B^1 (dashed line) and g_A (thick solid line).

III. ANOMALOUS MAGNETIC MOMENT

We are now ready to compute physical quantities to one loop order. Since we are interested in observed particles, the external legs, besides having on-shell momenta, always correspond to the observed zero modes $\psi^{(0)}$ and $A_\mu^{(0)}$ (the massive modes $\psi_i^{(1)}$, $A_\mu^{(1)}$ and $B_\mu^{(1)}$ have never been detected thus far, therefore m_f and m_b must be at least of order 1 TeV [5]). Further, integrals over internal momenta will be evaluated with a UV cutoff $\Lambda \sim m_f > m_b$, since the effective Lagrangian $L_{(4)}$ holds for momenta below the confining threshold only.

The EM one-loop contribution to the anomalous magnetic moment of leptons, $\Delta = (g-2)/2$, is represented by the three graphs in Fig. 4. From all three graphs we extract the form factor corresponding to the magnetic

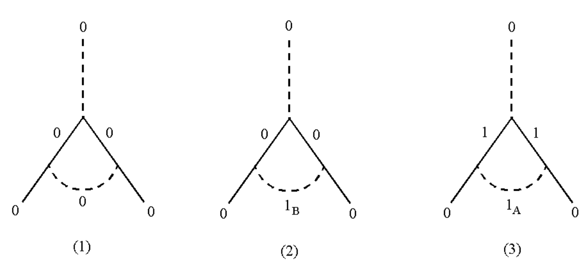

FIG. 4. Feynman graphs contributing to $g-2$.

component in the Gordon decomposition of the fermion current (see, *e.g.*, Ref. [12]) and obtain the following results: The first graph yields

$$\Delta_{(1)} = \frac{\alpha}{2\,\pi}\,\left[1+\mathcal{O}\left(\frac{m^2}{\Lambda^2}\right)\right]\ , \qquad (16)$$

where the first term is the standard (one-loop) correction and the second term vanishes when the cut-off $\Lambda\to\infty$; the second graph gives

$$\Delta_{(2)} = \frac{\alpha}{2\,\pi}\,\left(\frac{g_B^0}{e}\right)^2\,\frac{2\,m^2}{3\,m_b^2}\,\left[1+\mathcal{O}\left(\frac{m^2}{\Lambda^2}\right)\right]\ ; \qquad (17)$$

finally, from the third graph we obtain

$$\Delta_{(3)} = \frac{\alpha}{2\,\pi}\,\left(\frac{g_A}{e}\right)^2\,\frac{5\,m}{12\,m_f}\,\left[1+\mathcal{O}\left(\frac{\Lambda-m_b}{\Lambda}\right)\right]\ , \qquad (18)$$

which is convergent. We recall here that $\Lambda\sim m_f$ and notice that $\Delta_{(3)}$ becomes negligible for $m_b\ll m_f$ because of the dependence on the ratio m_b/m_f of the coupling constant g_A (see Fig. 3).

The measured anomalous magnetic moments of the electron and muon are in agreement with the SM predictions to a very high level of precision. Therefore, the second term in $\Delta_{(1)}$, which arises because of the cut-off $\Lambda\sim m_f<\infty$, and $\Delta_{(2)}$ and $\Delta_{(3)}$, which involve virtual $\psi^{(1)}$, $A_\mu^{(1)}$ and $B_\mu^{(1)}$, must be smaller than the experimental error in $(g-2)/2$ and this implies limits on the possible values of m_f and m_b or, equivalently, on the thickness of the brane-world. In particular, $(g-2)/2$ of the electron is measured with an error $\Delta_e\sim 4\cdot 10^{-12}$ [13,14]. Thus, from Eq. (16) with $m=m_e\sim 0.5\,\mathrm{MeV}$ and $\Lambda\sim m_f$ one has that $m_f>10\,\mathrm{GeV}$, which is however less restrictive than the constraint $m_f, m_b>1\,\mathrm{TeV}$. The latter constraint also renders the contribution of the second graph practically negligible, since the coupling constant $1/3<g_B^0/e<2/3$ and $\Delta_{(2)}$ is thus of the same order as the second term in $\Delta_{(1)}$. A stronger prediction instead comes from Eq. (18) if $m_b\sim m_f$ ($g_A/e\sim 1/3$), namely $m_f>1.3\cdot 10^7\,m_e\sim 7\,\mathrm{TeV}$. For the muon ($m=m_\mu\sim 100\,\mathrm{MeV}$), $(g-2)/2$ is measured with an error $\Delta_\mu\sim 10^{-9}$ [14]. Eq. (16) then implies $m_f>100\,\mathrm{GeV}$ and, from Eq. (18), $m_f>5\,\mathrm{TeV}$. Future experiments are expected to lower the error down to $\Delta_\mu=4\cdot 10^{-10}$ [15], which would imply a limit $m_f>10\,\mathrm{TeV}$.

The existence of $\psi^{(1)}$ and massive gauge bosons also gives new radiative corrections to the lepton ($\psi^{(0)}$) self-mass. Such terms, because of the longitudinal vector field contribution, have a leading divergence of the form $(\Lambda/m_b)^2\,\ln(\Lambda/m_f)$, which, in contrast to the usual QED case, is multiplied by a factor proportional to m_f ($\gg m_e,\ m_\mu$). Clearly, this implies that such contributions could be large. However, by suitably adjusting Λ, m_b and m_f, one can keep the corrections finite and small, reabsorbing them in the definition of the physical mass and the renormalization of external legs. This allows one to have a relatively large contribution to $g-2$ without affecting the mass of the light leptons.

Lastly, one may worry about weak corrections: again, in this case, the masses of the "excited" (W's and Z) bosons and fermions will appear [see, *e.g.*, our Eq. (18)] and the contributions will be comparable to our purely EM corrections, thus leading (barring improbable cancellations) to analogous results.

IV. CONCLUSIONS

In this letter we have considered a model for a thick brane-world of codimension one and included in the EM one-loop computation of the anomalous magnetic moment of leptons the contribution of virtual massive states living inside the brane. This allows us to put constraints on the possible mass of "excited" states in the form of lower limits of the order of 10 TeV. Several simplifying assumptions have been made, in particular we have just considered one massive mode both for leptons and the photon. The results we have obtained could be generalized to include more massive states (given a detailed model for the confinement) and the approach we have followed applied to other effects.

We finally observe that, given our construction of a thick brane-world, our results can be viewed as complementary to those derived from the inclusion of bulk gravitons [16], SM fields living in the bulk [17] or other extensions beyond the SM [18].

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